Konvergenz des Proximal–Punkt–Verfahrens für inkorrekt gestellte Optimalsteuerprobleme mit partiellen Differentialgleichungen

Dissertation

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Abstract:

The concept of multi-step prox-regularization over a sequence of grids with application to solving ill-posed variational inequalities was proposed and theoretically motivated by Kaplan and Tichatschke in 1997 in their paper "Prox-regularization and solution of ill-posed elliptic variational inequalities". The same paper is also concerned with the problem of partial regularization on a subspace. The relevant applications include, in particular, ill-posed optimal control problems, where the subspace is formed by the control variables in process space.

In the first part of this thesis we consider abstract linear-quadratic optimal control problem, formulated in general Hilbert spaces. We discuss general assumptions, concerning optimal control problem, and also conditions, under which this problem becomes ill-posed. Then two general multi-step partial prox-regularization schemes are considered. In first case the MSR-method is regarded with state equation, relaxed using penalty technique in accordance with Kaplan/Tichatschke's works. In the second case auxiliary problems of the MSR-method are formulated in a presumption that the state equation is fulfild strictly.

Main part of the thesis is dedicated to the analysis of a convergence of the approximate solutions of MSR-method's auxiliary problems to the element of the optimal set of the original problem. The question is: in what case the weakest convergence criteria for the finite elements approximation could be found? To reveal this, two specific optimal control problems with elliptic systems in the case of distributed control were studied.

First of them is well known Fuller's problem, for which an analytical solution with so called chattering regime could be found, and which is the base example for the ill-posed optimal control problems discussed here. We formulate the MSR-method for the Fuller's problem, in which the penalty error and the finite elements approximation error were taken into account. As a main result convergence criterion of the MSR-method was obtained, concatenating limit behavior of the penalty, regularization and discretization parameters.

In the last chapter we formulate another ill-posed optimal control problem, with distributed control in polygonal domain. A Poisson problem with mixed boundary conditions is now considered as state equation. Such a formulation of the problem seems to be a natural generalization of the classical Fuller's problem with the sec-

ond order ordinary differential equation to the case of two-dimensional domains. We reformulate the MSR-method to solve this new problem, and along with the approximation error, the computational errors are also taken into account. But now we do not use penalty technique, and form auxiliary problems with strict fulfilment of the state equation instead. This alternative approach gives us an opportunity to obtain, using Falk's proof technique, *weaker and thus more advantageous convergence criteria for the MSR-method.*

In the conclusion we present the results of numerical computations using the MSR-method for one certain problem, solution of which produces two-dimensional chattering regime in the square region.